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A QUANTITATIVE APPROACH TO BALANCING MANPOWER REQUIREMENTS AND --ETC(U)
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A QUANTITATIVE APPROACH TO BALANCING MANPOWER REQUIREMENTS AND PERSONNEL INVENTORIES

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A model for the Navy's Enlisted Personnel System was formulated to determine (rather than predict) an advancement and recruit input/training "policy" for each of the enlisted ratings. The formulation is intended to make a feasible determination based on the personnel system's structure, estimates of promotion eligible personnel, and the supply of recruits. A penalty function is used to measure the discrepancy between "requirements" and strength, by rating and pay grade, over time. Bounds on the strength will not permit a feasible solution to create an excessive shortage or overage, unless it is inherited from the beginning inventory. (over)

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The inventory is categorized by skill, period, pay grade, and time-in-grade. Also, the apprentice population (those in Pay Grades E-1--E-3), is categorized by period and time in service, but not by skill. "Strikers" are accounted for implicitly.

FOREWORD

This research and development was conducted in support of the development of a manpower requirements determination system for Navy enlisted personnel, and was performed under subproject Z0109-PN.07, Techniques for Balancing Manpower and Personnel. The purpose of this subproject, which terminated in FY78, was to explore quantitative methods for reconciling imbalances between manpower requirements and existing personnel inventories.

The model formulated in this report provides an explicit framework for viewing the relationship of manpower and personnel. As such, it will provide an experimental vehicle in helping to understand that complex relationship.

DONALD F. PARKER
COMMANDING OFFICER

SUMMARY

Problem

The Navy's force of enlisted personnel consists of about 450,000 individuals. Of these, roughly 100,000, or 22 percent, leave the system annually and must be replaced. Because budget and personnel ceilings prescribed by Congress must be strictly observed, the management of the Navy's manpower and personnel system is largely centralized.

Very broadly stated, the objectives of the enlisted manpower and personnel system are to enlist, train, maintain, and distribute personnel in accordance with demonstrated manpower requirements. These objectives must be attained within constraints established by the mandates of Congress and the policies of the Secretary of Defense, Secretary of the Navy, and Chief of Naval Operations.

Given this situation, it is not surprising that a mismatch often occurs between personnel available and manpower requirements. "Balancing," as referred to in the report, is the planning and actions taken to minimize this disparity or reduce it to an acceptable level.

Objective

The objective was to help define the "balancing" problem in precise terms. Variables that quantify the problem, especially from the manpower requirements determination viewpoint, were to be defined. These variables were to be evaluated for their influence on the system.

The specific objective was to develop a methodology in which advancement flow, training requirements, and recruit requirements are all subject to joint and collective determination. All current and future information would be used at each point in time to minimize the cost of failing to meet manpower requirements, while not violating various policy and law-of-motion constraints.

Approach

A model was formulated in terms of a linear programming framework within which the Navy's Enlisted Personnel System can be studied. The purpose of this model is to determine (rather than predict) the advancement, recruit input, and training requirements for each of the enlisted ratings. The formulation is intended to make a feasible determination based on the personnel system's structure and estimates of promotion-eligible personnel and the supply of recruits. A penalty function is used to measure the discrepancy between requirements and strength, by rate and rating, over time. Bounds on the strength will not permit a feasible solution to create an excessive shortage or overage, unless it is inherited from the beginning inventory or forced on the system by radical changes in requirements. The policy determination has a variable horizon, expected to be 5 years in practice.

Findings and Conclusions

The data required to implement this model consist of the enlisted inventory, which is available from the enlisted master record (EMR). Data analysis would be necessary to develop estimates of the continuation rates and promotion eligibility rates. Data processing to "feed" the model would have to encompass all ratings.

The major set of parameters, whose values are to be set by the user, is the cost function specifications (Z) which price the discrepancies (D). These parameters should assume different values in different applications of the model. That is, the correct set of values will depend on the purpose of the specific application at hand.

The slopes of the cost function (Z) imply a relative marginal utility of personnel, either above or below their respective authorizations. This is an area in which further research is needed, and from which many other studies in Total Force management could benefit. The dual solution to this linear program provides marginal prices on each constraint. These prices would give Navy manpower planners a specific value associated with raising or lowering manpower requirements (equivalently, authorizations), which, in the current solution, cannot be met. This logic can be used for adjusting requirements in response to inventory inelasticity.

While the linear goal programming model presented in this report simultaneously considers various aspects of personnel planning, it should be viewed as a point of departure for discussion of balancing methodologies--not as the definitive solution. The model does have the singular virtue of assessing the elasticity of personnel inventory changes over time in response to manpower requirements. The model will be used experimentally, to gain insight into the complex relationship between personnel resources and manpower requirements.

CONTENTS

INTRODUCTION

Problem

The Navy's Enlisted Manpower and Personnel System, which is largely centralized in the Washington, DC area, is responsible for maintaining a force of roughly 450,000 personnel. Of these, approximately 100,000 (or 22%) leave the system annually, and must be replaced. Internal flows are more numerous: Approximately 130,000 promotions are made within the petty officer force. This number includes about 70,000 advancements to third class, which implies a substantial formal training requirement. Planning goals and quotas for all these flows must be specified for each rate and rating, or about 600 categories.

The manpower requirements determination process is a large-scale decision-making process. Requirements are aggregated from the ship, squadron, and shore station levels by rate and rating. Requirements must also be determined for several eventualities, such as peacetime, mobilization, and wartime levels. Usually, unconstrained requirements exceed the manpower levels and budget permitted by Congress, and, in some cases, exceed the levels that can be filled by normal management of the personnel inventory. It is also interesting to note that the number of requirements listed in the billet file typically exceeds the number of billets themselves. Given this situation, it is not surprising that the match between personnel inventory and manpower requirements is often imperfect. Bringing this disparity to acceptable levels is the problem of "balancing."

Purpose

The objective was to help define the "balancing" problem in precise terms. Variables that quantify the problem, especially from the manpower requirements determination viewpoint, were to be defined. These variables were to be evaluated for their influence on the system.

The specific objective was to develop a methodology in which advancement flow, training requirements, and recruit requirements were all subject to joint and collective determination. All current and future information would be used at each point in time to minimize the cost of failing to meet manpower requirements, while not violating various policy and law-of-motion constraints.

Background

Organizational Aspects of Total Force Management

In reviewing some recent reports concerned with defense manpower, particularly Navy manpower, it became apparent that the balancing problem is a subset of the problems surrounding Total Force Management. Total Force Management is a concept under which all personnel resources available to a particular DoD service--active duty and reserve military, civilian, and contractor--are managed as a single resource; that is, with an eye to possible tradeoffs among classes of personnel or manpower. Under this concept, life cycle costs, personnel policies, and job requirements are jointly considered in an effort to arrive at an optimal force mix and deployment. Planning for and implementation of these concepts are certainly related to solutions of the balancing problem.

Two of these studies--the Defense Manpower Commission Report and the Salzer Report--are described below. Both of these studies agreed that Total Force Management is not a reality, due, in part, to fragmentation of responsibility for various key aspects of manpower management. This fragmentation in organization is subsequently reflected in

management processes and methodologies, especially those involved in the balancing problem.

Defense Manpower Commission Report. The Defense Manpower Commission (DMC, 1976), in its report to the President, concluded that "the Total Force policy of the Department of Defense is far from a reality and . . . the expectations of it may have been overstated." To remedy this situation, the Commission made the following recommendations:

- Define the manpower function to include all manpower life cycle activities and vest the responsibility for this function in one individual at each layer (of management) determined to be necessary.

- Grant manpower organizations the responsibility for all elements of the Total Force and for all life cycle functions. This will allow more uniform management of programming and budgeting for the Total Force. Further, each life cycle function should be managed on a Total Force basis.

Salzer Report. The Salzer Report (Salzer, 1976) provides an evaluation of the Navy's present manpower, personnel, and training management structure; proposed changes considered necessary; and a conceptual framework for integrating manpower and personnel management. This was seen to address the problems of balancing directly, especially the organizational aspects. In the summary, Salzer stated:

> The study found that different types of Navy manpower are controlled by separate organizations and that manpower management is not functionally integrated with the management of personnel or training. In short, the criticisms voiced by several Congressional Committees earlier in 1976 regarding the organizational fragmentation of the Navy's manpower management processes were correct. This constitutes an impediment to optimal implementation of the "Total Force" policy. Problems were also found to exist regarding methodologies for analysis of manpower, personnel, and training on an integrated basis.

The Salzer Report was a thorough study of the Navy's Manpower management organization. Salzer found, as explained above, that management processes and methodologies did not include analyses of manpower, personnel, and training on a Total Force integrated basis. This methodological deficiency is no doubt due to the fragmentation of organizational responsibility, which discourages, if not prevents, methodologies and management analysis of manpower and personnel issues on a collective or integrated basis. The quantitative approach presented in this report is directed at this problem specifically.

Current Methodologies

The balancing problem has been addressed by various Navy staff organizations, leading to various approaches and management processes. This section discusses some of these current practices and initiatives, showing the present context for the quantitative formulation developed in the following section.

Manpower Requirements Determination. The Navy's manpower requirements determination process is associated with a development known as the Navy Manpower Planning System (NAMPS). The primary objectives of this system (NAMPS, 1977) are to determine military and civilian manpower requirements, and to give management support to the analysis of these requirements throughout the Planning, Programming, and Budgeting System (PPBS) cycle. The methodologies being developed to support NAMPS include improved methods for obtaining Ship Manning Documents (SMD) and Squadron Manning Documents (SQMD), as well as for validating Shore Requirements (SHORESTAMPS). The intent is to be able to modify requirements as weapons and support systems are moved in and out of the force mix. Special effort is being made to relate manpower requirements to operational requirements, so that the effect of not programming certain manpower (billets) can be related to operational capability.

Authorization Management. One initiative that is closely related to the balancing problem is Authorization Management. In this management process, future distributable inventory is projected early enough in the PPBS cycle to affect which requirements are programmed (authorized). The approach is to determine the billets that are not likely to be filled in the near term because of inventory inelasticity and that, consequently, should not necessarily be funded. Although a methodology is being developed to support this process, there is no way to identify how many more resources would be required to fill the unfunded billets. Also, the importance of having each particular billet filled creates a cost structure on the inventory, as sponsors compete for the set of personnel projected to become available.

Personnel Inventory Management. Personnel inventory management of the enlisted force is supported by the Advancement, Strength, and Training Planning System (ADSTAP). The output and goals of the system are presented in the U.S. Navy Enlisted Force Management System annual report to ASD (M&RA) (White Book, 1976). The primary thrust of long-range planning presented in this report is in driving toward an ideal force structure, which is based on a utility measure of the enlisted inventory. The continuance behavior, advancement opportunity, and related parameters describing personnel flows were varied to obtain the "optimum" steady state force structure (Schmid, Hovey, & Mayberry, 1977). The result is "ideal" force, described by rate, rating, and length of service, which reflects the minimum cost per utile for the entire force.

The ADSTAP system supports enlisted force management, both on a day-to-day level and on a long-range planning basis. The computerized decision methodology embodied in ADSTAP ties together the training, recruitment, advancement, and strength planning actions into one place for reconciliation. The main inventory projection instrument in the ADSTAP System is known as the Force Structure Simulation Model (FAST) (Boller, 1974). The FAST model is closest to a Total Force Management tool, especially for 1- to 2-year planning horizons. FAST takes manpower authorizations as fixed goals to which the personnel inventory should be manipulated via personnel policy. For example, when future requirements at a given pay grade increase, followed by a decrease, advancements to this pay grade and all lower ones exhibit an especially unstable behavior. What may appear as "mild" billet fluctuations become "severe" advancement flow fluctuations. The organizational fragmentation described earlier heretofore has prevented consideration of alternative methodologies that address the balancing problem in a model such as FAST.

METHODOLOGY

Quantitative Formulation for Balancing

This section describes a specific quantitative formulation of a mathematical programming problem that could form the basis for methodological developments that explicitly address the balancing problem. The context for this model is one of long-range planning; for example, over the 5-year defense plan horizon. Although the manpower requirements are assumed as a known, some degree of uncertainty could be accommodated. The model strives to prescribe the "best" set of recruit inputs, allocation to ratings of new petty officers, and advancements within the petty officer force, at the rate and rating level, over the planning horizon.

Synopsis of the Formulation

To accommodate this formulation, many independent variables, parameters, and data values will be defined. The independent variables are those whose values are to be determined by the model, corresponding to an optimum solution. Generally, independent variables are also policy variables that Navy manpower managers can control. Prime examples are the numbers of personnel to be advanced, by rate and rating, over the planning horizon. Data values are values that are to be estimated, usually on a statistical basis from historical analysis. Examples of these are continuation rates and test passer availability rates. Determination of data values is time consuming but not conceptually difficult. Parameters fall in a grey area, in that they are determined by the user. Sensitivity analysis with the model itself, however, might influence their choice. Examples of parameters are the time-in-grade (TIG) requirements, bounds on shortages and overages allowed in meeting requirements, and penalty rates on shortages and overages.

Constraint equations implement a law of motion for the enlisted force structure that assume bottom level entry, with vacancy driven advancement. The inventory is stratified by rate, rating, and a TIG index. This latter dimension permits explicit TIG policies to be imposed and also increases the precision of loss estimation. Personnel are assumed to flow from outside the system into an apprentice pool that is not rating specific. After an appropriate "waiting" period, inventory flows into E-4, the beginning of the petty officer force, in each rating. These flows are independent variables.

Most constraints deal with intra-rating flows, but one important constraint relates ratings. This constraint requires the authorized end strength in each rate and year to be met at the All Navy level. Although this constraint could be relaxed if desired, it will usually cause overages in response to uncontrolled shortages in a given rate in an attempt to "use" all the authorized end-strength. The model makes these inter-rating adjustments in an optimum manner.

The objective function is a penalty function that places a cost on shortages and overages in an attempt to meet manpower requirements. This, then, drives promotions so as to meet requirements. Determination of the objective function parameters is undoubtedly the most difficult aspect of this formulation; that is, one must assign, in arbitrary relative units, the "cost" associated with being undermanned or overmanned (costs will generally differ). These can be differential by rate, rating, and planning year. In addition, upper and lower bounds on the manning, which must be met to obtain a feasible solution, must also be specified.

These parameters have not been given serious study in the past, due to the dichotomy between personnel and manpower management explained previously. If any quantitative methodologies are to be developed that address the balancing problem, this subject must receive serious consideration. Some preliminary suggestions are discussed in a later section.

Relationship to Existing Methodologies

Existing methods for determining manpower requirements (Ship Manning Documents (SMDs), *Squadron Manning Documents* (SQMDs), and the emerging SHORESTAMPS program) do so independently of personnel inventory considerations. These values, after some adjustment for career path feasibility and budget limitations, are used by personnel inventory managers as production targets.

The FAST model for personnel inventory projection uses these targets to develop advancement flow. Each rating is treated independently for each planning year in isolation from the future years. Adjustments to advancements are made to satisfy the inter-rating constraint of meeting authorized end strength by rate, again in isolation of future year's requirements. Personnel flow into the petty officer force from the unrated apprentice population is controlled primarily by the short-term needs in each rating and training capacity constraints imposed externally.

The fundamental distinction between the existing methodology and the approach outlined in this report is that advancement flow, training requirements, and recruit requirements would all be subject to joint and collective determination using all current and future information at each point in time. *The object of the model is to minimize the cost of failing to meet manpower* requirements while not violating various policy and law-of-motion constraints. The two primary differences are in the use of future requirements to anticipate certain overage and shortage situations, and in the explicit "pricing" of unfilled or overfilled billets.

Currently, this latter situation occurs implicitly, through the Navy Manning Plan (NMP). This plan allocates available personnel to available billets in the short term, based on a priority score of billets given by commanding officers. This becomes an allocation of scarce resources after the fact, instead of a long-range planning tool. NMP, however, would be needed in any case to accommodate the inevitable unforeseen changes and minor unavoidable discrepancies that arise. It is currently used more extensively in the implicit setting of priorities than it might be. The current management initiative, which is seen to achieve a better reconciliation of this condition, is Authorization Management. The latter would, when operational, anticipate the results of Navy Manning Plans 1 year in the future. At this point, there would still be some budget flexibility to make changes in inventory production that might ease the anticipated situation.

The techniques to be introduced here are not new, especially in the area of manpower planning. Neither is it claimed that the formulation is definitive or necessarily unique in the balancing problem. The prior section has, however, shown that current planning methodologies do not provide for balancing the value of meeting manpower requirements against the costs associated with doing so. The specific approach discussed here is a candidate that is intended to stimulate consideration of this and possibly other approaches to this problem.

MODEL FORMULATION

Mathematical Assumptions

This model will be stated in terms of a linear programming framework. The specific intent of the model is to determine (rather than predict) the advancement, recruit input, and training requirements for each of the enlisted ratings. The formulation is intended to make a feasible determination based on the personnel system's structure and estimates of promotion-eligible personnel and the supply of recruits. A penalty function is used to measure the discrepancy between "requirements" and strength, by rate and rating, over time. Bounds on the strength will not permit a feasible solution to create an excessive shortage or overage, unless it is inherited from the beginning inventory or forced on the system by radical changes in requirements. The policy determination has a variable horizon, expected to be 5 years in practice.

Constraint Equations and Personnel Flows

The following constants are in universal use for what follows:

S = Number of skills (ratings), expected to be approximately 100.

H = Problem horizon (period), expected to be the 5 years of the FYDP.

G = Number of pay grades per skill, expected to be the six petty officer grades.

T = Maximum time-in-grade (TIG) required for advancement (about 4 years).

Constraints--Inventory Flows

The inventory is categorized by skill, period, pay grade, and TIG. Also, the apprentice population (i.e., those in Pay Grades E-1--E-3) are categorized by period and TIG, but not by skill. "Strikers" can be accounted for implicitly. The inventory of period h refers to the onboard inventory at the beginning of period h, and periods are indexed beginning with the number 1. The period 1 inventory is hence a beginning inventory, and is assumed to be known. Inventory is treated as an independent variable, and its value is computed with the following equations.

Let

$I(s,h,g,t)$ = Inventory in skill s, at the beginning of period h, in pay grade g with time in grade index t.

This variable is dimensioned $S \times (H+1) \times G \times (T+1)$. The TIG index t is a waiting index; that is, the number of waiting periods required until a promotion can occur.

Let

$IA(h,t)$ = Inventory in the apprentice groups, at the beginning of period h, with TIG index t.

This variable is dimensioned $(H+1) \times (T+1)$.

See Figure 1 for a visual representation of inventory flows.

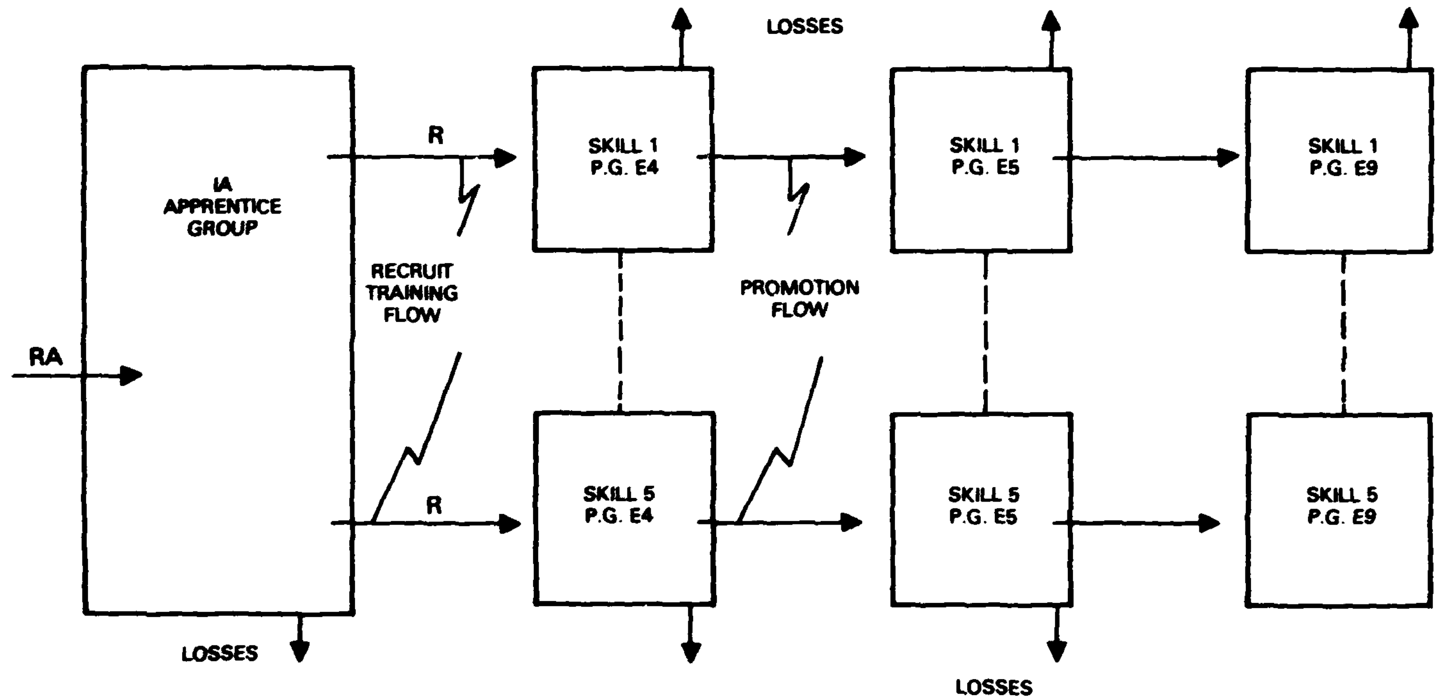

Figure 1. Conceptual personnel flow modelled by constraint equations.

In equation (constraint) form, these inventory flows are

$$I(s,h = 1,g,t) = B(s,g,t), \text{ given beginning inventory}$$

$$IA(h = 1,t) = BA(t), \text{ given beginning inventory} \tag{1}$$

where

$$g = 1, \ldots, G,\ t = 0, \ldots, T$$

$$\begin{aligned} I(s,h+1,g,t) = {} & ID(t < T)\, C(s,h,g,t+1)\, I(s,h,g,t+1) \\ & + ID(t=0)\, C(s,h,g,t) \{I(s,h,g,t) - P(s,h,g)\, ID(g < G)\} \\ & + ID(g > 1)\, C(s,h,g,t)\, L(s,h,g,t)\, P(s,h,g-1) \\ & + ID(g=1)\, C(s,h,g,t)\, L(s,h,g,t)\, R(s,h) \end{aligned} \tag{2}$$

$$\begin{aligned} IA(h+1,t) = {} & ID(t < T)\, CA(h,t+1)\, IA(h,t+1) \\ & + ID(t=0)\, CA(h,t) \{IA(h,t) - \sum_{s=1}^{S} R(s,h)\} \\ & + LA(h,t)\, RA(h) \end{aligned} \tag{3}$$

where

$$h = 1,2, \ldots, H,\ s = 1, \ldots, S,\ t = 0, \ldots, T,\ g = 1, \ldots, G.$$

Terms used in the above equations are defined as follows:

B and BA are beginning inventory, assumed given as data. Equation (1) sets the inventory in period 1.

$$ID(t < T) = \begin{cases} 1 & \text{if } t < T, \text{ e.g., } 0,1, \ldots, T-1 \\ 0 & \text{if } t = T \end{cases}$$

A similar definition holds for ID(t=0), . . . etc. This is simply a device for including or excluding terms in the constraints, without having to write special equations.

C(s,h,g,t) is a <u>continuation</u> fraction. This <u>data</u> represent the fraction of the inventory I(s,h,g,t) that is expected to continue into period h+1 (i.e., survive period h). Values are determined from data, and are <u>not</u> independent variables. CA(h,t) is a continuation fraction for IA(h,t), and is assumed to be data input.

P(s,h,g) are the promotions <u>from</u> skill s, pay grade g, during period h, into pay grade g+1. The range of indices are s = 1, . . . ,S; h - 1, . . . ,H; g = 1, . . . ,G - 1 (no promotions from E-9). This is an <u>independent</u> variable. Its value will be adjusted by the linear programming algorithm to determine an "optimum" value. As such, this represents a part

of the promotion policy being determined by the model. Note that promotions from grade g represent a personnel loss to grade g, and a gain to grade g+1.

L(s,h,g,t) is a policy variable to be input by the user. Its interpretation is as follows: L(s,h,g,t) P(s,h,g) represents those being promoted during period h, in skill s, into pay grade g, that will enter with TIG index t, s = 1, . . . ,S; h = 1, . . . ,H; g = 1, . . . ,G; t = 0,1, . . . ,T. The values for L(s,h,g,t) must be nonnegative fractions that sum to 1.0 over the TIG dimension t = 0, . . . ,T. LA(h,t), similarly defined, is a policy variable associated with new recruits entering the system. These variables, L(s,h,g,t) and LA(h,t), enable the user to distribute P(s,h,g) (promotions), R(s,h) (apprentices entering rating S), and RA(h) (recruits) along TIG dimension.

The TIG index t has the following interpretation: Having an index of t means t periods must elapse before a promotion can occur. Promotions are constrained to come from the inventory (I or IA) with t = 0. Conceptually, an individual entering a new grade (i.e., being promoted) is assigned an index, T (say 2), and each period thereafter, his index decreases by 1, until reaching 0. Those with t = 0, if not promoted, remain in the t = 0 population.

Use of the variable L, together with the TIG index t, allow the user to enforce a policy regarding TIG requirements for promotions. This would include apprentice time prior to eligibility for entry into a rating. The variable LA is similarly defined for this purpose.

R(s,h) are recruit trainees and strikers to be brought into rating s, during period h. RA(h) are the number of recruit apprentices (recruits) to be brought into the system during the period h. These quantities are cast as <u>independent</u> variables, and imply training requirements and recruiting goals, respectively, for the system. Although these variables could become parameters to be specified exogeneously, this would result in an inferior solution to the total force problem. If, in fact, these variables are constrained (for example, by training plant facilities and production smoothing, and by the availability of qualified personnel), then such constraints should be explicitly taken into account.

Equations (2) and (3) are similar, describing the inventory flows. Equation (2) states that next period's inventory consists of those who continue in service with a 1-year increase in TIG, less promotions into the grade above, plus promotions from the grade below, plus input from the apprentice population in the case of pay grade E-4. Equation (3) states that the apprentice population is decreased by advancements to the rated population, and increased by new recruits. These are standard accounting equations for personnel inventory management.

<u>Constraints--Resource Limitations</u>

There are, of course, further constraints on the variables introduced above. One such constraint is the eligibility of personnel for promotion. These are <u>inequality</u> constraints that require promotions to be less than or equal to the eligible population from which they will be drawn.

$$P(s,h,g) \leq E(s,h,g)\, I(s,h,g,t = 0) \qquad (4)$$

where

s = 1, . . . ,S; h - 1, . . . ,H; and g = 1, . . . ,G - 1; and

$$\sum_{s=1}^{S} R(s,h) \leq EA(h)\ IA(h,t=0) \tag{5}$$

where

$h = 1, \ldots, H.$

All terms above have been defined, except E and EA. E(s,h,g) is the fraction of inventory in skill s, during period h, in pay grade g, time in grade index 0, that will be eligible to advance. EA is similarly defined, and applies to the apprentice population. These variables represent <u>data</u> to be input. Note that promotions are assumed to be from the TIG index = 0 population, consistent with t being a <u>waiting</u> index.

$$P(s,h,g) \geq F(h,g)\ I(s,h,g,t=0) \tag{6}$$

where

$s = 1, \ldots, S; h = 1, \ldots, H;$ and $g = 1, \ldots, G - 1.$

This constraint enforces a token promotion policy that says that a fraction F(h,g) of I(s,h,g,t,t=0) must be promoted. The value of F is a policy parameter input by the user.

<u>Constraints--Requirement Targets</u>

The goal of the model is to determine advancements that maintain personnel inventories at a level satisfying requirements. Since not all requirements can be met, we define a deviation, D, from requirements as given, and assess a penalty for nonzero values. The following equation (constraint) defines D.

$$D(s,h,g) = A(s,h,g) - \sum_{t=0}^{T} I(s,h+1,g,t) \tag{7}$$

where

$s = 1, \ldots, S;\ h = 1, \ldots, H;$ and $g = 1, \ldots, G.$

Here A(s,h,g) is the desired strength (requirements or authorizations) in skill s, pay grade g, at the <u>end</u> of period h. Values for A are <u>data</u> input.

Deviation, D, is simply the discrepancy between on-board strength and desired strength. It can be positive or negative, and is an <u>independent</u> variable. Its value is set by the linear programming algorithm in a fashion that minimizes the overall penalty discussed below, subject to Equation (7). Note that the desired strength, A, could be unconstrained requirements, authorizations, or "what if" values of strength.

The following constraint says that all Navy end strength by pay grade and period is fixed and that any promotion policy must account for this. If the data A(s,h,g) in Equation (7) is authorizations, then we have

$$\sum_{s=1}^{S} D(s,h,g) = 0 \tag{8}$$

where

$$h = 1, \ldots, H; \text{ and } g = 1, \ldots, G.$$

If the sum $\sum_{s=1}^{S} A(s,h,g)$ differs from the end strength authorized by Congress, then this difference would replace the right-hand side in Equation (8). There is also a total enlisted end strength, which essentially forms a constraint on the apprentice population, as follows:

$$\sum_{t=0}^{T} IA(h+1,T) = ES(h) - \sum_{s=1}^{S} \sum_{g=1}^{G} A(s,h,g). \tag{9}$$

In this equation, ES(h) is the total All Navy end strength authorized by Congress at the end of period h. This is a <u>data</u> input.

Constraints--Critical Manning Levels

These constraints are bounds on the deviations, D, that are judged as feasible. This is stated so as to avoid excessive overages and shortages, particularly in critical skills and pay grades.

$$U(s,h,g) \leq D(s,h,g) \leq U'(s,h,g) \tag{10}$$

where

$$s = 1, \ldots, S; \; h = 1, \ldots, H; \text{ and } g = 1, \ldots, G.$$

Here, U and U' will be parameters input by the user, and could be fractions of A(s,h,g), representing the tolerance for overages and shortages, respectively, in that rating. Note $U \leq 0$, $U' \geq 0$.

Objective Function

The objective function consists of assigning a cost to each nonzero deviation, D, and then summing up all costs. These cost functions cannot be linear, or undesirable results would be imposed by the model. Instead, a convex cost is desired. This is one whose costs increase even faster as the deviation grows in absolute value. To keep the formulation within the bounds of linear programming without compromising the objective of the problem, a functional form (Figure 2) is assumed.

This convex cost function Z is a function of the deviation, D. Conceptually, the above penalty (cost) would be computed for each value of s, h, and g, and added together; that is,

$$Z = \sum_{s=1}^{S} \sum_{h=1}^{H} \sum_{g=1}^{G} Z(s,h,g).$$

The objective of this formulation will be to minimize the following function:

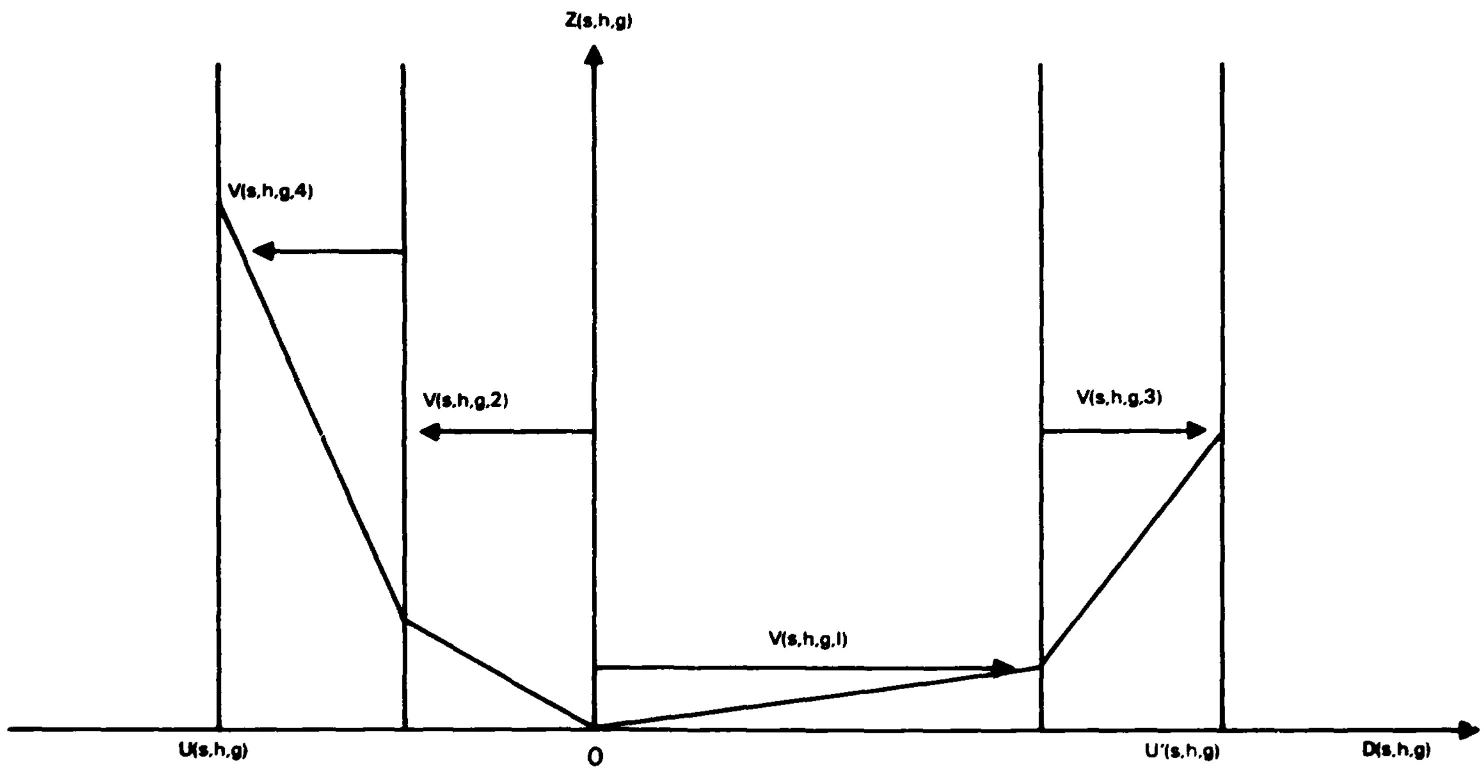

Figure 2. Convex objective function.

An optimum solution is, in this context, one that minimizes the value of Z. Note that the user would have control over the relative shape of each cost function Z(s,h,g), so that one can set the relative criticality of different ratings, pay grades, and time periods. The choice of a convex cost is necessary to avoid all shortages (or overages) being taken up in the "cheapest" rating. To implement the nonlinear cost in a linear programming format, each deviation will be actually replaced by four variables, as follows:

$$D(s,h,g) = D(s,h,g,1) + D(s,h,g,3) - D(s,h,g,2) - D(s,h,g,4)$$

$$D(s,h,g,p) \geq 0 \tag{11}$$

where

$$s = 1, \ldots, S;\ h = 1, \ldots, H;\ g = 1, \ldots, G; \text{ and } p = 1, \ldots, 4.$$

The cost function can then be written as

$$Z(s,h,g) = \sum_{i=1}^{4} Q(s,h,g,i)\, D(s,h,g,i).$$

Here, Q(s,h,g,i) is a nonnegative cost coefficient (a parameter) input by the user. As a result of this transformation, constraint Equations (7) and (8), containing deviation, D, will then have the Equation (11) replacing D. Constraint Equation (10) then becomes

$$0 \leq D(s,h,g,p) \leq V(s,h,g,p) \tag{12}$$

where

$$s = 1, \ldots, S;\ h = 1, \ldots, H;\ g = 1, \ldots, G \text{ and } p = 1, \ldots, 4.$$

The values of the limits V(s,h,g,p) are <u>data input</u>, and are shown in the above figure.

The final form of this problem is then to minimize Z subject to the constraint Equations (1) through (10), deviation Equations (11) and (12), and the nonnegativity constraints,

$$P(s,h,g) \geq 0 \ (g=1, \ldots, G1)$$

$$I(s,h,g,t) \geq 0$$

$$IA(h,t) \geq 0$$

$$D(s,h,g,p) \geq 0$$

$$R(s,h,) \geq 0$$

$$RA(h) \geq 0$$

where

$$h = 1, \ldots, H;\ s = 1, \ldots, S;\ g = 1, \ldots, G;\ t = 0, \ldots, T;\ p = 1, \ldots, 4; \text{ and where}$$

$$Z = \sum_{s=1}^{S} \sum_{h=1}^{H} \sum_{g=1}^{G} \sum_{i=1}^{4} Q(s,h,g,i)\, D(s,h,g,i).$$

An optimal solution will yield values for the above independent variables. P(s,h,g) will be the number of promotions in each rating and pay grade, for each period to the horizon. I(s,h,g,t) and IA(h,t) will be the resulting inventory in each rating, pay grade, and period from this promotion policy. R(s,h) will specify how many apprentices should enter the rated petty officer force during each period, and RA(h) will specify how many new recruits should enter the apprentice population each period. The deviation, D, measures the difference between projected strength and a desired strength each period. When a problem is found to be infeasible, as is expected to happen, constraints can be selectively relaxed until a feasible solution is found.

FINDINGS

Parameter and Data Requirements

The data required to implement this model consists of enlisted inventory, stratified by rate, rating, and time in grade (TIG), which are readily available from the Enlisted Master File. Some data analysis would be necessary to develop estimates of the continuation rates and promotion eligibility rates used in Equations (2) through (10). The necessary source data from which these estimates can be made is also available. Authorizations to be used as production targets are routinely available, and would be subject to "what if" variations.

The major set of parameters, whose values are to be set by the user, is the cost function specifications (Z) which price the discrepancies (D). These data consist of cost rates (i.e., the slopes of the convex cost function), and the bounds (V) on acceptable overages and shortages for a feasible solution. These parameters should be thought of as properly assumed different values in different applications of the model; that is, the correct set of values will depend upon the purpose of the specific application at hand.

Since this parameter selection can be one of the more difficult aspects of using the model, the following suggestions are made. Note that positive values of the discrepancy (D) correspond to authorizations exceeding personnel, or shortages, which could be limited by a percentage of the authorization itself. This would correspond to setting the sum

$$V(s,h,g,1) + V(s,h,g,3)$$

to this upper limit. $V(s,h,g,1)$ could then be determined as a lesser percentage of authorizations, corresponding to the break between being marginally below strength and critically undermanned. This, of course, then determines $V(s,h,g,3)$.

Negative values of the discrepancy, (D), correspond to personnel exceeding authorizations, which occurs in individual ratings. The upper limit on these deviations,

$$V(s,h,g,2) + V(s,h,g,4),$$

is provided by the "true" requirements, from which authorizations are usually derived, and would not normally be exceeded.

The slopes of the cost function Z imply a relative marginal utility to personnel, either above or below their respective authorizations. This is the precise area in which further research is called for, and from which many other studies in Total Force Management could benefit. Some work in relative utility (not however marginal) has been done, and could be used to determine the penalty coefficients. Another source of data for these parameters is the Navy Manning Plan. Implicit in this program are the relative criticalities of unfilled billets, which should lead to a true marginal utility of billets by rate and rating. These marginal utilities may not reflect the priorities of Navy manpower planners for future years, but would reflect current priorites set by the operational community.

The dimension of future time is also required in the determination of these objective function parameters. One common technique in dealing with planning for periods with uncertainty is to discount returns and penalties from future periods. This would be accomplished by simply reducing the penalty function (Z) by a multiplicative discount factor for each successive year. This is, admittedly, arbitrary as further study may in fact suggest equal or reverse weighting, giving the planning years more weight.

Applications

Objective Function Units

Interpretation of objective function units always creates a special problem. There is, of course, no physical interpretation of these units; but this should not present any real problem. The units are only a device for seeking alternative solutions, and the solutions themselves should be examined. That is, to evaluate an "optimal" solution, one should analyze the results by carefully comparing them against the otherwise nominal 5-year plan. The planned advancement flow over time, discrepancies over time, recruiting requirements, and implications for school input should all be studied to learn what the optimal solution has attempted to do. Similar quantities for the alternative plans should then be compared to the optimal solution and evaluated from a practical viewpoint.

Applications to Manpower Requirements

The dual solution to this linear program provides marginal prices on each constraint. These prices would give Navy manpower planners a specific value associated with raising or lowering manpower requirements (equivalently authorizations) that, in the current solution, cannot be met.

This provides a logical basis for adjusting requirements in response to inventory inelasticity. Those ratings providing the greatest marginal gain might be chosen for requirements adjustment first. Such an adjustment would have an effect throughout the system, since severe discrepancies in one rating cause discrepancies of the opposite sign in other ones so as to satisfy All Navy constraints. A new optimal solution would then be found, and the evaluation repeated until manpower inelasticity inhibited any further adjustments.

CONCLUSIONS

The data required to implement this model consists of enlisted inventory, which is available from the enlisted master record (EMR). Some data analysis would be necessary to develop estimates of the continuation rates and promotion eligibility rates.

The major set of parameters, whose values are to be set by the user, is the cost function specifications (Z) that price the discrepancies (D). These parameters should assume different values in different applications of the model; that is, the correct set of values will depend on the purpose of the specific application at hand.

The slopes of the cost function (Z) imply a relative marginal utility of personnel, either above or below their respective authorizations. This is the precise area in which further research is called for, and from which many other studies in Total Force management could benefit. The dual solution to this linear program provides marginal prices on each constraint. These prices would give Navy manpower planners a specific value associated with raising or lowering manpower requirements (equivalently authorizations), which, in the current solution, cannot be met. This logic can be used for adjusting requirements in response to inventory inelasticity.

While the linear programming model presented in this report simultaneously considers various aspects of personnel planning, it should be viewed as a point of departure for discussion of balancing methodologies--not as the definitive solution. The model described in this report should help in understanding the "balancing" problem and the complex relationships between personnel resources and manpower requirements. Because the model is a point of departure, it will be used as an experimental vehicle to aid in the development of solutions for related operational problems.

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